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MAXIMAL MONOTONICITY AND BIFURCATION FROM THE CONTINUOUS SPECTR--ETC(U)

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Tassilo Küpper and Jürgen Weyer

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53706

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MAXIMAL MONOTONICITY AND BIFURCATION FROM THE CONTINUOUS SPECTRUM

Tassilo Küpper[*1,2] and Jürgen Weyer[†3]

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ABSTRACT

Consider a class of nonlinear eigenvalue problems of the form

$$T^*\Phi Tu - F(u) = \lambda u,$$

where T is a linear closed operator and where Φ and F are nonlinear gradient operators satisfying $\Phi(0) = F(0) = 0$, and thus $u \equiv 0$ is a solution for all values of λ. The equation is studied with particular emphasis to bifurcation from the trivial line of solutions, including bifurcation from the continuous spectrum of the linearized problem.

For the special case that Φ = identity (i.e. $T^*\Phi T$ is positive selfadjoint) it has recently been shown that the lowest point of the spectrum of the linearized problem is a bifurcation point under suitable conditions on F. The proof makes extensive use of the decomposition of positive selfadjoint operators. In this paper we show that these results carry over to the nonlinear case, provided that Φ is maximal cyclically monotone.

The results are illustrated by nonlinear ordinary differential equations on unbounded intervals where the linearized problem has a purely continuous spectrum. Due to the general form of the leading part nonlinearities in the highest occurring derivatives are permitted.

AMS(MOS) Subject Classifications: 34B15, 47E05, 47H05

Key Words: Bifurcation, Continuous Spectrum, Maximal Monotonicity, Ordinary Differential Equations

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*Department of Mathematics and Mathematics Research Center, University of Wisconsin-Madison.
†Universidad de Santiago de Chile, Casilla 5659, Santiago 2, Chile.

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SIGNIFICANCE AND EXPLANATION

Several problems of mathematical physics such as nonlinear Schrödinger - or Klein-Gordon-Equations lead to nonlinear eigenvalue problems whose linear part has a purely continuous spectrum; thus classical methods of bifurcation theory cannot be applied. However, often the linear part is a positive selfadjoint operator admitting a special decomposition which can be used to establish the existence of nonlinear solutions. In this article we first generalize this method and show that the same results hold if the selfadjoint operator is replaced by a maximal cyclically monotone operator. Maximal cyclically monotone operators as introduced by T. Rockafellar are the nonlinear analogue of the selfadjoint operator. For example, maximal cyclically monotone operators arise in elasticity problems if a nonlinear elasticity law instead of Hooke's law is used.

To illustrate the influence of a strong nonlinearity in the leading part of the equation the abstract results are applied to second order differential equations on unbounded intervals.

MAXIMAL MONOTONICITY AND BIFURCATION FROM THE CONTINUOUS SPECTRUM

Tassilo Küpper*,[1,2] and Jürgen Weyer†,[3]

1. INTRODUCTION

Several problems of mathematical physics as nonlinear Schrödinger - or Klein-Gordon-Equations for example lead to nonlinear eigenvalue problems of the abstract form

$$T^*Tu - F(u) = \lambda u \; . \tag{1.1}$$

Here T denotes a linear and closed operator in an Hilbert space H and F a nonlinearity of higher order terms. Hence, the problem is normalized so that $u \equiv 0$ is a (trivial) solution for all values of λ and it is the subject of bifurcation theory to find nontrivial solutions branching from the trivial line of solutions.

The physical background of the problems suggests to permit operators T such that the linear part T^*T of the equation has a purely continuous spectrum. For instance, this happens to be the case for nonlinear ordinary or partial differential equations on unbounded domains as can already be seen from the one dimensional problem.

$$-u'' - r(x)|u|^\sigma u = \lambda u \quad (u \in H := L^2(\mathbb{R})) \; . \tag{1.2}$$

*Department of Mathematics and Mathematics Research Center, University of Wisconsin-Madison.
†Universidad de Santiago de Chile, Casilla 5659, Santiago 2, Chile.

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To fit this problem into the abstract setting choose $Tu = u'$ and $F(u) = r(x)|u|^{\sigma}u$ with $\sigma > 0$ and r nonnegative. However, classical bifurcation theory does not apply to (1.2) since the linear part $T^*Tu = -u''$ has a purely continuous spectrum which covers the positive halfaxis $[0,\infty)$.

Several recent papers [1,2,3,4,5] have dealt with equations of the type (1.1) and shown that the lowest point of the (continuous) spectrum of T^*T is in fact a bifurcation point under suitable conditions on the nonlinearity F. It has been shown in [6,7] that the nonlinear analogue of these operators are operators of the type $T^*\Phi T$ where Φ denotes a maximal cyclically monotone operator. In this case $T^*\Phi T$ is maximal cyclically monotone, too, and many properties of T^*T carry over to $T^*\Phi T$.

We shall prove in Theorem 2.1 that these properties already suffice to obtain existence and bifurcation in the case of a rather general nonlinearity Φ.

In the applications to differential equations this means that nonlinearities in the derivatives can be permitted. For example $-u''$ can be replaced by the more general expression $-g(u')'$ provided g is strictly increasing. Nonlinearities of this kind for instance occur if a nonlinear elasiticity law instead of Hooke's law is used.

As a specific example for such a nonlinearity Kauderer [8] mentions the torsion vibrations of a cylinder which are described by the nonlinear eigenvalue problem (see also [9],[10]):

$$-u'' \pm (u'^3)' = \lambda u \,, \quad u(0) = u(1) = 0 \,.$$

Here we are particularly interested to consider ordinary differential equations on unbounded intervals as applications of Theorem 2.1. Due to the unboundedness of the interval the maximal monotonicity is no longer an easy consequence of abstract theorems. By using an approximation technique we are

still able to prove that operators of the kind $-g(u')'$ are maximal cyclically monotone even on unbounded intervals. In order to treat rapidly growing nonlinearities we have to make another approximation by bounded nonlinearities. Applying Theorem 2.1 to the auxiliary problem we even get bifurcation for the general case.

2. THE ABSTRACT THEOREM

Let $(H; \langle\ ,\ \rangle; \|\ \|)$ be a real Hilbert space and assume that $T : D(T) \subseteq H \to H$ is a linear closed and densely defined operator. Then we consider the abstract nonlinear eigenvalue problem

$$T^*\Phi Tu - F(u) = \lambda u \tag{2.1}$$

where the nonlinear operators Φ and F satisfy the following hypotheses:

(A1) The operators $\Phi : H \to H$ and $T^*\Phi T : D(T^*\Phi T) \subseteq H \to H$ are maximal cyclically monotone and satisfy $\Phi(0) = 0$; $D(T^*\Phi T) \subseteq D(T^*T)$ and for some constant $M > 0$

$$\|T^*Tu\| \leq M\|T^*\Phi Tu\| \quad (u \in D(T^*\Phi T)) \, . \tag{2.2}$$

In addition $\Phi_{|D(T)}$ is the gradient of a C^1-functional $\phi : D(T) \to \mathbb{R}$ with $\phi(0) = 0$ and

$$\|Tu\|^2 \leq 2\,\phi(u) \qquad (u \in D(T)) \, .$$

(A2) The operator $F : D(T) \to H$ is the gradient of a C^1-functional $f : D(T) \to \mathbb{R}$ with $f(0) = 0$ such that for some constants $k > 0$ and $\alpha \in [0,2)$, $\beta > 0$ with $\alpha + \beta > 2$:

$$0 \leq \langle F(u),u\rangle \leq k\|Tu\|^{\alpha}\|u\|^{\beta} \qquad (u \in D(T)) \, .$$

Further, $F(u_n)$ converges to $F(u)$ for any sequence $u_n \in D(T)$ such that $u_n \rightharpoonup u \in D(T)$ and $Tu_n \rightharpoonup Tu$.

(A3) There exists a constant $\tau > 2$ such that for all $u \in D(T)$ and all $t \geq 1$:

$$\phi(tu) \leq t^{\tau}\phi(u) \quad , \quad f(tu) \geq t^{\tau}f(u) \, .$$

To prove the existence of nontrivial solutions of (2.1) we consider this nonlinear eigenvalue problem as the Euler-Lagrange equation of the

functional $\psi(u) := \phi(Tu) - f(u)$ which will be minimized on spheres $S(c) := \{u \in D(\phi T) / \|u\| = c\}$.

<u>THEOREM 2.1</u>: Assume that (A1), (A2) and (A3) hold and that for some $c > 0$

$$a_c := \inf\{\psi(u)/u \in S(c)\} < 0 \; . \tag{2.4}$$

Then there exists a solution $(u_c, \lambda_c) \in S_c \cap D(T^*\Phi T) \times (-\infty, 0)$ of (2.1) such that $\psi(u_c) = a_c$ and

$$-k(2k/\tau)^{\alpha/(2-\alpha)} c^{2(\alpha+\beta-2)/(2-\alpha)} \le \lambda_c \le \tau a_c / c^2 \tag{2.5}$$

$$\|T u_c\| \le (2kc^{\beta}/\tau)^{1/(2-\alpha)} \; . \tag{2.6}$$

<u>Remark</u>: (Bifurcation): If there exists an interval $(0, c_0)$ such that (2.4) holds for all $c \in (0, c_0)$ then $\lambda = 0$ is a bifurcation point in the sense that for $c = \|u_c\| \to 0$:

$$\lambda_c \to 0 \quad \text{and} \quad \|T^*T u_c\| \to 0 \; .$$

<u>Remark</u>: The special version of Theorem 2.1 with $\Phi(u) = u$ has been established by Stuart [3] who also showed that the weak continuity of f can be replaced by conditions which permit wider range of applications. We note that these results also carry over in the case of general Φ in a similar way as in Theorem 2.1 but omit the details for the sake of brevity and refer to Grade [11].

In the proof of Theorem 2.1 we need the following result on maximal monotone operators:

<u>LEMMA 2.2</u>: Let $T^*\Phi T$ be a maximal cyclically monotone operator and $u \in D(\Phi T)$. If $\langle \Phi Tu, Tv \rangle = \langle w, v \rangle$ for all $v \in D(T)$ and some $w \in H$ then $u \in D(T^*\Phi T)$ and $w = T^*\Phi Tu$.

<u>Proof of Lemma 2.2</u>: Due to the monotonicity of Φ we conclude for all $x \in D(T^*\Phi T)$ that $v := x - u \in D(T)$ and :

$$0 \le \langle \Phi Tx - \Phi Tu, Tv \rangle = \langle \Phi Tx, Tv \rangle - \langle \Phi Tu, Tv \rangle$$

$$= \langle \Phi Tx, Tv \rangle - \langle w, v \rangle \; .$$

Since $x \in D(T^*\Phi T)$ this yields $\langle T^*\Phi(Tx),v\rangle - \langle w,v\rangle \geq 0$, and $\langle T^*\Phi Tx - w, x-u\rangle \geq 0$, hence $u \in D(T^*\Phi T)$ and $w = T^*\Phi Tu$ by the maximality of $T^*\Phi T$.

Proof of Theorem 2.1: From (A3) it follows that

$$\frac{1}{t}\,[\phi((1+t)u) - \phi(u)] \leq \frac{1}{t}\,[(1+t)^\tau - 1]\phi(u) \qquad (t > 0)\ .$$

Taking the limit $t \to +0$ and making use of the formula of L'Hospital we conclude for $u \in D(T)$:

$$(\Phi(u),u) \leq \tau\,\phi(u)\ . \tag{2.7}$$

By the same method we get

$$(F(u),u) \geq \tau f(u)\ . \tag{2.8}$$

Now, by (2.4) there exists a sequence $\{u_n\} \in S(c)$ such that $\lim_{n\to\infty} \psi(u_n) = a < 0$ and $\psi(u_n) < 0$, hence $f(u_n) > \phi(Tu_n) \geq \|Tu_n\|^2/2$ by (A1). Using (2.8) we get $\tau f(u_n) \leq \langle F(u_n),u_n\rangle \leq k\|Tu_n\|^\alpha \|u_n\|^\beta$ and hence $\|Tu_n\|^{2-\alpha} \leq (2k/\tau)\|u_n\|^\beta$. Because $\|u_n\| = c$ the sequences $\{u_n\}$ and $\{Tu_n\}$ are bounded and there exist subsequences such that $u_n \rightharpoonup u_c$ and $Tu_n \rightharpoonup v$ in H. Since T is linear and closed we have $u_c \in D(T)$, $Tu_c = v$ and $u_c \in D(\Phi T)$. Further, the functional ϕ is weakly lower semi-continuous (Rockafellar [11]), so that $\phi(Tu_c) \leq \liminf_{n\to\infty} \phi(Tu_n)$. Using $f(u_c) = \lim_{n\to\infty} f(u_n)$ we get $\psi(u_c) \leq \lim_{n\to\infty} \phi(u_n) = a < 0 = \psi(0)$, hence $u_c \neq 0$. Suppose now that $\|u_c\| < c$ and define $t := c/\|u_c\| > 1$. Then $tu_c \in S(c)$ and $\psi(tu_c) = \phi(tTu_c) - f(tu_c) \leq t^\tau(\phi(Tu_c) - f(u_c)) < \psi(u_c) = a$ in contradiction to $\psi(t\,u_c) \geq a$. Because of $\|u_c\| \leq \limsup_{n\to\infty} \|u_n\| = c$ we have $u_c \in S(c)$.

By the Euler-Lagrange principle there exists a Lagrange multiplier $\lambda_c \in \mathbb{R}$ such that for all $v \in D(T)$

$$\lambda_c\ \langle u_c,v\rangle = \psi'(u_c)v = \langle \Phi Tu_c, Tv\rangle - \langle F(u_c),v\rangle\ .$$

Lemma 2.2 finally implies $u_c \in D(T^*\Phi T)$ and $T^*\Phi T u_c - F(u_c) = \lambda_c u_c$. While the formula (2.6) has already been derived in the proof, (2.5) is now an easy consequence of (A2) and (2.7), (2.8).

The bifurcation result follows from the continuity of $F : D(T) \to H$ and the estimates:

$$\|T^* T u_c\|/M \le |\lambda_c| \|u_c\| + \|F(u_c)\| .$$

Hence, the convergence $\lambda_c \to 0$ is a consequence of (2.5).

3. APPLICATIONS

The abstract results are now used to study nonlinear ordinary differential equations of second order on an unbounded interval Ω which we choose without restriction as $\Omega = (0,\infty)$. In Ω we consider the differential equation:

$$- (u' + |u'|^p u')' - r(x)|u|^\sigma u = \lambda u \tag{3.1}$$

where p and σ are positive exponents and r satisfies for some $t > 0$:

(R) $\quad r \in L^\infty(0,\infty)$, $\lim\limits_{x\to\infty} r(x) = 0$

$\quad \operatorname*{ess\,sup}\limits_{x>0} x^t r(x) < \infty$

$\quad \operatorname{ess\,inf} (1+x)^t r(x) > 0$.

As boundary conditions we impose either Dirichlet boundary conditions

$$u \in H := L^2(0,\infty) \text{ , } u(0) = 0 \tag{3.2}$$

or Neumann boundary conditions at 0 :

$$u \in H := L^2(0,\infty) \text{ , } u'(0) = 0 \text{ .} \tag{3.3}$$

To fit this problem into the abstract setting we define the following operators:

$$Lu := -u' \text{ , } D(L) := \{u \in H \ / \ u' \in H\}$$

$$\Phi(u) := g(u) := u + |u|^p u \text{ , } D(\Phi) := \{u \in H \ / \ g(u) \in H\} \text{ .}$$

It is well known that $D(L^*) = \{u \in H \ / \ u' \in H \text{ , } u(0) = 0\}$, $L^*u = u'$ and $(L^*)^* = L$. In the case of Dirichlet boundary condition we choose $T = L^*$. In the case of the Neumann condition we choose $T = L$. It is further known (see Stuart [4,5]) that the relation $F(u) := r|u|^\sigma u$ defines a continuous operator $F : D(T) \to H$.

Using these notations we see that the problem (3.1) with (3.2) or (3.3) can be written in the form

$$T^*\Phi Tu - F(u) = \lambda u \text{ .} \tag{3.4}$$

The corresponding linearized problem has a purely continuous spectrum covering the positive half axis. We shall see that bifurcation from the lowest point of the continuous spectrum occurs only for suitable constants p and σ.

<u>THEOREM 3.1</u>: Dirichlet problem (3.1), (3.2).

Suppose (R) holds. Then:

(i) There is no bifurcation if $\sigma > \max\{0,4-2t\}$.

(ii) There is bifurcation at $\lambda = 0$ if $t \in (0,2)$ and $p < \sigma < 4-2t$.

<u>THEOREM 3.2</u>: Neumann problem (3.1), (3.3).

Suppose (R) holds. Then:

(i) There is no bifurcation if $\sigma > \max\{2,4-2t\}$.

(ii) There is bifurcation at $\lambda = 0$ if

$$p < \sigma < \max\{2,4-2t\} .$$

Since it is our main aim to study the influence of the leading nonlinearity $T^*\Phi T$ we have restricted our consideration to a nonlinearity F and a potential $f(u) = (\sigma+2)^{-1}\int_0^\infty r(x)|u|^{\sigma+2}(x)dx$ which have already been shown to satisfy (A2) and (A3). Weaker assumptions on F are possible but lead to more lengthy formulations and have been omitted for that reason (see Grade [12], Stuart [4,5]).

Hence to prove Theorems 3.1, 3.2 with the aid of Theorem 2.1 we can essentially concentrate on the verification of (A1). This is done in two steps. First we show that both operators $T^*\Phi T$ $(T \in \{L,L^*\})$ are maximal cyclically monotone. Second we deal with the difficulty that the operator $\Phi(u) := g(u)$ is not defined on the whole space H if g grows too strongly at infinity.

3.1 Maximality of $-(g(u'))'$ on $(0,\infty)$.

It was shown in [6,7] that Φ is maximal cyclically monotone and, due to $\lim_{|x|\to\infty} |g(x)/x| \geq \delta > 0$, surjective. Since the operator $A := T^*\Phi T$ $(T \in \{L, L^*\})$ is obviously cyclically monotone, it remains to show that A is maximal. To prove maximality of A it suffices to show by [6,7] that either T or T^* has a bounded inverse, or, more general, that the range of T is closed. It is easy to see that due to the unboundedness of the interval neither $T = L$ nor $T = L^*$ have a closed range so that [6,7] does not apply immediately. However, the maximality of A can still be established by approximating A by maximal cyclically monotone operators on bounded domains.

THEOREM 3.3: Suppose $g \in C^1(\mathbb{R})$, $g(0) = 0$ and $g'(x) \geq \delta > 0$ for all $x \in \mathbb{R}$. Then the operator $Au := -g(u')'$ is maximal cyclically monotone on each of the following domain of definition:

(i) $D(A) = \{u \in L^2(0,\infty)/u', u'' \in L^2(0,\infty)\ ,\ u(0) = 0\}$

(ii) $D(A) = \{u \in L^2(0,\infty)/u', u'' \in L^2(0,\infty)\ ,\ u'(0) = 0\}$.

Proof of Theorem 3.1: To prove maximality of A we have to show that the differential equation

$$-g(u'(x))' + u(x) = h(x) \tag{3.5}$$

has for each $h \in L^2(0,\infty)$ a solution $u \in L^2(0,\infty)$ satisfying the choosen boundary condition. We first study the differential equation (3.5) in the Hilbert space $H_n := L^2(0,n)$ with the usual norm $\|\ \|_n$ and inner product $\langle\ ,\ \rangle_n$. In addition we consider the Banach space $(C_0[0,n], \|\ \|_{\infty,n})$ of continuous functions on the interval $[0,n]$. In H_n we define the operators

$$D(L_n) := \{u \in H_n \ /\ u' \in H_n\}\ ,\ L_n u := -u'$$

$$D(L_n^*) := \{u \in H_n \ /\ u' \in H_n\ ,\ u(0) = u(n) = 0\}\ ,\ L_n^* u = u'$$

$$D(\Phi_n) := \{u \in H_n \ /\ g(u) \in H_n\}\ ,\ \Phi_n(u) = g(u)\ .$$

From the arguments of [6] it follows that Φ_n is maximal cyclically monotone. Further it is well known that $(L_n^*)^* = L_n$ and both L_n and L_n^* have a closed range and their kernels are contained in $D(\Phi_n)$:

$$N(L_n^*) = \{u \in D(L_n^*) \mathbin{/} L_n^* u = 0\} = \{0\} \subseteq D(\Phi_n)$$

$$N(L_n) = \{u \in D(L_n) \mathbin{/} L_n u = 0\} = \{u \in H_n \mathbin{/} u \equiv \text{const. a.e.}\} \subseteq D(\Phi_n) .$$

Since Φ_n is strictly monotone it is coercive. Now by [6,7] $A_n := T_n^* \Phi_n T_n$ is maximal cyclically monotone for $T_n \in \{L_n, L_n^*\}$ and each $n \in \mathbb{N}$. Consequently, there exists a solution $u_n \in H_n$ satisfying

$$-g(u_n')' + u_n(x) = h_{|(0,n)} \tag{3.6}$$

and either $u_n(0) = u_n(n) = 0$ or $u_n'(0) = u_n'(n) = 0$.

We now show that $\{u_n\}$ converges to a solution u of (3.5) satisfying $u(0) = 0$ or $u'(0) = 0$, respectively. Multiplication of (3.6) by u_n gives $\langle g(u_n'), u_n'\rangle + \|u_n\|_n^2 = \langle h,u_n\rangle_n \le \|u_n\|_n \|h\|$. Using $g' \ge \delta$ we obtain that $\|u_n\|_n$ and $\|u_n\|_n^2$ are bounded. From the differential equation we see that $g(u_n')$ is weakly differentiable. Since g is C^1 and strictly increasing we also have that u_n' is weakly differentiable and that $g(u_n')' = g'(u_n')u_n''$ holds. Multiplying (3.6) with $-u_n''$ we conclude that $\|u_n''\|_n$ is bounded:

$$\delta \cdot \|u_n''\|_n^2 \le \langle g'(u_n')u_n'' ,u_n''\rangle_n + \|u_n'\|_n^2 = - \langle h,u_n''\rangle_n \le \|h\| \, \|u_n''\|_n .$$

The functions $u_n, u_n', u_n'' \in L^2(0,n)$ can be extended by 0 to bounded functions in $L^2(0,\infty)$ and it is easy to see that these extended functions which we again denote by u_n, u_n' and u_n'' converge weakly in $L^2(0,\infty)$; precisely: $u_n \rightharpoonup u$, $u_n' \rightharpoonup v$, $u_n'' \rightharpoonup w$. Using integration by parts it can be shown that u and v are differentiable and satisfy $u' = v$ and $v' = w$.

We finally show that this function u satisfies the differential equation (in each bounded interval $(0,k)$), as well as the boundary condition.

Fix k. By Sobolev's Lemma we conclude that $\|u_n\|_{\infty,k}$ and $\|u'_n\|_{\infty,k}$ are uniformly bounded (for $n > k$). By the estimates

$$|u_n^{(i)}(x+\varepsilon) - u_n^{(i)}(x)| \le \int_x^{x+\varepsilon} |u_n^{(i+1)}| \le \varepsilon\|u_n^{(i+1)}\|_{\infty,k} \le \varepsilon \text{ const.} \quad (i = 0,1)$$

we have that the functions $\{u_n\}$ and $\{u'_n\}$ are equicontinuous in $(0,k)$. Now, by Arzela-Ascoli's theorem the sequences $\{u_n\}$ and $\{u'_n\}$ converge in $C[0,k]$ to u and u' respectively. Consequently we have for any weakly differentiable w satisfying $w(0) = w(k) = 0$:

$$\left|-\int_0^k g(u'_n(x))'w(x)+g(u'(x))w'(x)dx\right| = \left|\int_0^k (g(u'_n(x)) - g(u'(x)))w'(x)dx\right| \le$$

$$\le \|g(u'_n) - g(u')\|_{\infty,n} \cdot \int_0^k |w'(x)|dx \underset{n\to\infty}{\to} 0 .$$

Making use of (3.6) we conclude $\int_0^k g(u'(x))w'(x)dx = \int_0^k (h(x) - u(x))w(x)dx$. Because w is arbitrary, $g(u'(x))$ is differentiable and u satisfies the differential equation $-g(u')' + u = h$ in $(0,k)$.

Since all functions u_n satisfy the boundary condition at 0 the same is true for u. Further we note that $u \in L^2(0,\infty)$ since $(I+A)^{-1}$ is a contraction: $\|u\|^2 = \|(I+A)^{-1}h\|^2 \le \|h\|^2$. Hence u solves the operator equation $T^*\Phi Tu + u = h$ which proves that $A = T^*\Phi T$ is maximal monotone.

Finally we have to show that the domain of $A = T^*\Phi T$ is given by the formula (i) or (ii) of Theorem 3.3. Because u satisfies the desired boundary conditions at 0, it is sufficient to show that u' is absolutely continuous and $u'' \in L^2(0,\infty)$ if and only if $g(u')$ is absolutely continuous and $g(u')' \in L^2(0,\infty)$. Because of the regularity conditions on g the function $g(u')$ is absolutely continuous if and only if u' is. The

relations $|u''| \leq \frac{1}{\delta}|g(u')'|$ and $|g(u')'| \leq (\sup_{x\in[0,\infty)} |g'(u'(x))|) \cdot |u''|$ yield the equivalence of $u'' \in L^2$ and $g(u')' \in L^2$.

3.2 An Auxiliary Problem.

Theorem 2.1 does not immediately apply to the problem (3.4) since the operator $\Phi(u) = g(u) = u + |u|^p u$ is not defined on the whole space H. For that reason we first consider an auxiliary problem which arises from (3.1) by a truncation of g and which coincides with (3.1) for "small" solutions:

$$-\tilde{g}(u')' - r(x)|u|^\sigma u = \lambda u \tag{3.7}$$

$$u \in D(L) \quad \text{or} \quad u \in D(L^*) . \tag{3.8}$$

Here $\tilde{g}$ is determined for some $x_0 > 0$ by :

$$\tilde{g}(x) := \begin{cases} g(x) & \text{for } |x| \leq |x_0| \\ g'(x_0)(x-x_0) + g(x_0) & \text{for } x > x_0 \\ +g'(x_0)(x+x_0) - g(x_0) & \text{for } x < x_0 . \end{cases}$$

It is obvious that $\tilde{g} \in C^1(\mathbb{R})$, $\tilde{g}'(x) \geq 1$ and $|\tilde{g}(x)| \leq \alpha|x|$ with $\alpha := \max\{g(x_0)/x_0, g'(x_0)\}$. The operator $\tilde{\Phi}(u) := \tilde{g}(u)$ is defined on H and a maximal cyclically monotone potential operator whose potential is given by $\phi(u) := \int_0^1 \int_0^\infty g(su(x))u(x)dx\, ds$.

We now show that Theorem 2.1 applies for $T \in \{L, L^*\}$ to the problem

$$T^*\tilde{\Phi}Tu - F(u) = \lambda u . \tag{3.9}$$

By Theorem 3.3 $T^*\tilde{\Phi}T$ is maximal cyclically monotone, too. Hence, (A1) is satisfied with Φ,ϕ replaced by $\tilde{\Phi},\tilde{\phi}$. An easy calculation shows that (A3) holds for $\tilde{\phi}$ with $\tau := p + 2$ if $p \leq \sigma$. Further, we note that (A2) only concerns the nonlinear operator F. For simplicity we have chosen the hypotheses on r and σ in the same way as in [4,5]. Hence, we refer to [4,5] for the verification of (A2). Finally, (2.4) is fulfilled since a straightforward calculation yields for $\tilde{\psi}(u) := \|\tilde{\phi}(Tu)\|^2/2 - f(u)$:

$$\inf\{\tilde{\psi}(u) / u \in S(c)\} \leq \tilde{\psi}(w_\alpha) < 0 . \tag{3.10}$$

Here we have chosen $w_\alpha(x) := 2c\,\alpha^{3/2}xe^{-\alpha x} \in D(\tilde{\Phi}L^*)$ in the case of Dirichlet boundary conditions and $w_\alpha(x) := c(2\alpha)^{1/2}e^{-\alpha x} \in D(\tilde{\Phi}L)$ in the case of Neumann boundary conditions. In both cases w_α satisfies $\|w_\alpha\| = c$ and $\tilde{\psi}(w_\alpha) < 0$ if $\alpha < 0$ is sufficiently small and $\sigma < 4 - 2t$ resp. $\sigma < \max\{2,4-2t\}$.

By Theorem 2.1 there exists a solution $(\tilde{u}_c, \tilde{\lambda}_c) \in D(T^*\tilde{\Phi}T) \times (-\infty,0)$ of (3.4) such that $\|\tilde{u}_c\| = c$ and for some positive constants $K_1,\gamma_1,K_2,\gamma_2$:

$$\|\tilde{u}_c'\| \le K_1c^{\gamma 1} \; , \; |\tilde{\lambda}_c| \le K_2c^{\gamma 2}$$

$$\|\tilde{u}_c''\| \le |\tilde{\lambda}_c| \, \|\tilde{u}_c\| + \|F(\tilde{u}_c)\| \; .$$

Applying the inequality $|u^2(x)| \le 2\|u\| \, \|u'\|$ $(u \in D(L))$ to $\tilde{u}_c'$ we obtain

$$\max |\tilde{u}_c'(x)| \le (2\|\tilde{u}_c'\| \, \|\tilde{u}_c''\|)^{1/2} \; .$$

Since F is continuous, there exists a constant $c(x_0) > 0$ such that $\max |\tilde{u}_c'(x)| \le x_0$ for all* $c \in (0,c(x_0))$ and consequently $\tilde{\Phi}(\tilde{u}_c') = \Phi(\tilde{u}_c')$. Hence $(\tilde{u}_c,\lambda_c)$ is a solution of (3.4), too.

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Consider a class of nonlinear eigenvalue problems of the form

$$T^*\Phi Tu - F(u) = \lambda u,$$

where T is a linear closed operator and where Φ and F are nonlinear gradient operators satisfying $\Phi(0) = F(0) = 0$, and thus $u \equiv 0$ is a solution for all values of λ. The equation is studied with particular

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emphasis to bifurcation from the trivial line of solutions, including bifurcation from the continuous spectrum of the linearized problem.

For the special case that Φ = identity (i.e. $T^*\Phi T$ is positive selfadjoint) it has recently been shown that the lowest point of the spectrum of the linearized problem is a bifurcation point under suitable conditions on F. The proof makes extensive use of the decomposition of positive selfadjoint operators. In this paper we show that these results carry over to the nonlinear case, provided that Φ is maximal cyclically monotone.

The results are illustrated by nonlinear ordinary differential equations on unbounded intervals where the linearized problem has a purely continuous spectrum. Due to the general form of the leading part nonlinearities in the highest occurring derivatives are permitted.

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